(Uncertainty)^2

How uncertain is your uncertainty budget?

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Abstract

Uncertainty budgets are receiving increasing attention not only from accredited laboratories, but from the metrology community at large. Often uncertainties are given with three or more significant digits, indicating that the uncertainty has been determined to within one percent or better, but how accurately can uncertainty be estimated?

The paper explores the capabilities of common statistical techniques as well as some of the tools given in the Guide to the Expression of Uncertainty in Measurement. Based on the integrity of these fundamental uncertainty building blocks, the uncertainty of sample uncertainty budgets is estimated.

Introduction

The International Vocabulary of Basic and General Terms in Metrology[1] (VIM) defines *measurement uncertainty* as a "parameter, associated with the result of a measurement, that characterizes the dispersion of the values that could reasonably be attributed to the measurand". It also defines *measurand* as a “particular quantity subject to measurement”.

As laboratories and metrologists become more and more interested in measurement uncertainty, and explore various ways to estimate the measurement uncertainty, the question comes to mind: “How well can we know the measurement uncertainty?”

Sometimes we see measurement uncertainty indicated with 3 - 4 - 5 or more significant digits, but can we really estimate the measurement uncertainty this well? That is the subject of this paper.

Basic Assumptions

This paper attempts to quantify the limitations of the GUM[2] method of estimating uncertainty. In order to do this, a number of assumptions have to be made.

The first assumption is that the analysis of the measuring process is generally sound, i.e. the measurement equation used is correct and the major uncertainty contributors have all been successfully identified and properly taken into account.

The second assumption is that all data used in type A estimates originate from well-designed experiments and that the significance of the variation seen in the experiments is understood.

The third assumption is that all type B estimates are based on sound engineering judgement made by knowledgeable persons understanding the measurement process in question.

If one or more of these assumptions are not met, any resemblance between the estimated uncertainty and the uncertainty of the measuring process analyzed is purely coincidental.

Uncertainty of type A evaluations

There are two main sources of uncertainty for type A evaluations. The first depends on the design of the experiment for the type A evaluation. For the evaluation to be meaningful, the experiment has to resemble the measuring process it is supposed to represent.

A short term repeatability study performed under controlled conditions in a laboratory environment by measuring parts that have been marked to ensure the participants measure in the same spot every time is not a good representation of a measuring process that takes place on the shop floor, measuring parts in random orientations as they come off a hot production machine.

But even if the experiment is well designed, there are still some limitations to how well a standard deviation can be estimated from a limited sample. Annex E.4 of GUM[2] addresses this issue and gives the following table:

Number of observations, n	$\sigma\left[s(\bar{q})\right] / \sigma(\bar{q})$ (Percent)
2	76
3	52
4	42
5	36
10	24
20	16
30	13
50	10

Table 1: *The standard deviation of the experimental standard deviation of the mean of n independent observations of a normally distributed random variable, relative to the standard deviation of that mean. From GUM*[2]

The table shows the relative uncertainty of the experimental standard deviation of the mean of a normally distributed random variable determined by a limited number of observations as a function of the number of observations.

The same relationship is true for the experimental standard deviation of the normally distributed random variable itself.

The table indicates that a standard deviation determined by 10 observations will be known within +/- 48 % at a 95% confidence level (2 standard deviations).

A standard deviation determined by 30 observations, which is a lot of observations by most peoples standards, will be known only to within +/- 26 % at a 95% confidence level (2 standard deviations).

It is surprising to most people how uncertain an experimental determination of the standard deviation of a variable is, even using relatively large data sets.

Uncertainty of type B evaluations

For type B evaluations, there are two sources of uncertainty in the estimate:

1. Uncertainty of the estimated variation limits
2. Uncertainty of the assumed distribution.

Both depend on the amount of information available about the parameter being evaluated.

Example: ID/OD Comparator Measurement

A 85 mm ring gage is calibrated on an ID/OD comparator. A stack of gageblocks of the same nominal size as the ring is used to "master" the comparator.

Measurement Conditions

The conditions are as follows:

- Temperature in the laboratory is 19.5ºC - 20.5ºC.
- The temperature difference between the master gage block stack and the measuring object is estimated to be less than 0.1ºC.
- The part is made of steel.
- The master gage blocks are made of steel.
- The operator is trained and familiar with the wringing of gage blocks and the use of the comparator.

Uncertainty Contributors

The following contributors are considered:

- Difference in expansion due to difference in thermal expansion coefficient (CTE)
- Difference in expansion due to difference in temperature
- Comparator Resolution
- Uncertainty of length of master gage block stack
- Repeatability including the effect of wringing

Quantifying Uncertainty Contributors

The following table summarizes the uncertainty contributors.

Contributor	A/B	Distribution	Number of measurements	Variation Limit (+/-)	Standard Uncertainty
Difference in CTE	B	U-shaped	-	0.05 µm	0.035 µm
Difference in temperature	B	U-shaped	-	0.09 µm	0.065 µm
Resolution	B	Resolution	-	0.01 µm	0.003 µm
Master Uncertainty	B	Normal	-	0.12 µm	0.06 µm
Repeatability	A	-	30	-	0.04 µm
Combined Standard Uncertainty					0.10 µm
Expanded Uncertainty (k=2)					0.20 µm

Table 2: *Summary of uncertainty estimation for calibration of 85 mm ring using ID/OD comparator.*

Uncertainty of the uncertainty contributors

For the type B evaluations there are two sources of uncertainty in each estimate, the variation limits and the assumed distribution.

For the difference in CTE the variation limits are based on two assumptions; that the difference in thermal expansion coefficients between the master and the ring is 1.1 µm/m/ºC and that the average temperature of the two is within 19.5ºC - 20.5ºC.

Although there presumably is good temperature control in the laboratory, the range may well be 19.4ºC - 20.4ºC or 19.6ºC - 20.6ºC, which is a variation of +/- 20 %. We will assume this variation for this variable.

The distribution is assumed to be a U-shaped distribution, which has a conversion factor of 0.7 (1/√2). If the distribution was instead assumed to be rectangular, which has a conversion factor of 0.6 (1/√3) it would mean a change in the conversion factor of 14 %

For the difference in temperature the variation limits are based on two assumptions; that the nominal thermal expansion coefficients for the master and the ring is 11 µm/m/ºC and that the temperature difference between the two is within 0.1ºC.

Although there presumably is good temperature control in the laboratory, the difference may well be within 0.08ºC or 0.12ºC, which is a variation of +/- 20 %. We will assume this variation for this variable.

The distribution is assumed to be a U-shaped distribution, which has a conversion factor of 0.7 (1/√2). If the distribution was instead assumed to be rectangular, which has a conversion factor of 0.6 (1/√3) it would mean a change in the conversion factor of 14 %

The resolution is the only variable, for which we know the contribution with virtual certainty, since it is "mechanically" determined.

For the uncertainty of the master gage block stack our calibration supplier has provided us with a certificate, which states an uncertainty of 0.12 µm for the gage blocks. But the supplier may be better or worse than estimated. We will assume that the "true" uncertainty is in the range of 0.10 µm - 0.14 µm, which is a variation of +/- 17 %.

We will assume that since our calibration supplier has taken several contributors into account and since they probably are independent, that his uncertainty follows a normal distribution, according to the central limit theorem.

For the repeatability study, since we used 30 reading to estimate the standard deviation, the uncertainty of the standard deviation is +/- 26 % according to table 1.

The following table summarizes the limits of the assumed uncertainty in the uncertainty estimates.

Contributor	Percentage	Distribution	Original Standard Uncertainty	"Uncertainty of Uncertainty"
Limit of CTE	20 %	Rectangular	0.035 µm	0.0042 µm
Distribution for CTE	14 %	Rectangular	0.035 µm	0.003 µm
Limit of temperature difference	20 %	Rectangular	0.065 µm	0.0078 µm
Distribution for temperature difference	14 %	Rectangular	0.065 µm	0.0055 µm
Master uncertainty	17 %	Normal	0.06 µm	0.005 µm
Experimental Repeatability	26 %	Normal	0.04 µm	0.0052 µm
Combined standard "Uncertainty of Uncertainty"				0.013 µm
Expanded "Uncertainty of Uncertainty"				0.026 µm

Table 3: *Summary of uncertainty of uncertainty estimate for calibration of 85 mm ring using an ID/OD comparator.*

Conclusion

The above example assumes that all estimates made are best estimates and that they are independent of each other, such that some are too high and some are too low. For this reason we get an uncertainty of the uncertainty of 13 % (0.026 µm on 0.20 µm), even though we have uncertainties of up to 26 % on one estimate and 20 % on the estimate of our major contributor.

In most cases where all the estimates have been made by the same person, they will tend to all be either high or low, based on whether the person is making a conservative or optimistic estimate. In that case the estimate may be even further off.

Additionally, in the example there are several contributors of approximately the same magnitude. In many measurements there is one major contributor. In that cases, the uncertainty of the uncertainty is almost exclusively dependent on how well this one contributor is estimated.

However, as can reasonably be implied from the example, in most situations an uncertainty estimate is only reliable to within 10 - 20 %, even if the person estimating is knowledgeable about the measurement process in question, has made a correct analysis of the measurement process and has access to reliable information.

References

1. International Vocabulary of Basic and General Terms in Metrology.BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML., 1995

2. Guide to the Expression of Uncertainty In Measurement. BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML., 1995